CHAPTER 4 MOTION IN TWO AND THREE DIMENSIONS

Select odd-numbered solutions, marked with a dagger (†), appear in the Student Solutions Manual, available for purchase. Answers to all solutions below are underscored.

4-1. (a) Total displacement is (3.2 + 2.6) km 45° E of N + 4.5 km 50° W of N = $\underline{\text{7.0 km}}$, $\underline{\text{5° E of N}}$ (see Problem 3, Chapter 3)

(b) Avg vel = $\Delta\mathbf{r}/\Delta t$ = 7.0 km/1.25 h, 5° E of N = $\underline{\text{5.6 km/h, 5° E of N}}$

(c) $$\text{Avg speed} = \frac{\text{total distance}}{\text{time}} = \frac{3.2 + 2.6 + 4.5\ \text{km}}{1.25\ \text{h}} = \underline{8.24\ \text{km/h}}$$

4-2. $$\text{avg speed} = \frac{\text{distance}}{\text{time}} = \frac{\pi \times r}{1/2\ \text{yr}} = \frac{\pi \times 1.50 \times 10^{11}\ \text{m}}{1/2 \times (365.25 \times 24 \times 3600)\ \text{s}} = \underline{29.9\ \text{km/s}}$$

$$|\,\text{Avg vel}\,| = \left|\frac{\Delta\mathbf{r}}{\Delta t}\right| = \frac{2 \times r}{1/2\ \text{yr}} = \frac{2 \times 1.50 \times 10^{11}\ \text{m}}{1/2 \times 3.16 \times 10^{7}\ \text{s}} = \underline{19.0\ \text{km/s}}$$

4-3. 171 km/h = 47.5 m/s

If the bullet is fired at an angle θ, its position vector is given by $\mathbf{r}_{\text{bul}} = 366\sin\theta t\ \hat{\mathbf{i}} + 366\cos\theta t\ \hat{\mathbf{j}}$

For the bird, $\mathbf{r}_{\text{bird}} = (47.5)t\ \hat{\mathbf{i}} + 30\ \hat{\mathbf{j}}$

For the bird to be hit $\mathbf{r}_{\text{bul}} = \mathbf{r}_{\text{bird}}$.

Comparing the x-components, we have $366\sin\theta = 47.5$ $\theta = \sin^{-1}(47.5/366) = 7.45°$

Then $d = 30\tan\theta = 30\tan 7.45° = \underline{3.93\ \text{m}}$.

Thus the hunter must aim 3.93 m ahead of the bird in order to hit the bird.

4-4. At $t = 0$, the velocity is $\mathbf{v}$ = 30 km/h N, or

$\mathbf{v} = \underline{0\mathbf{i} + (30\ \text{km/h})\mathbf{j}}$. At $t = 10$ s,

$\mathbf{v}$ = 30 km/h @ 45° W of N, or

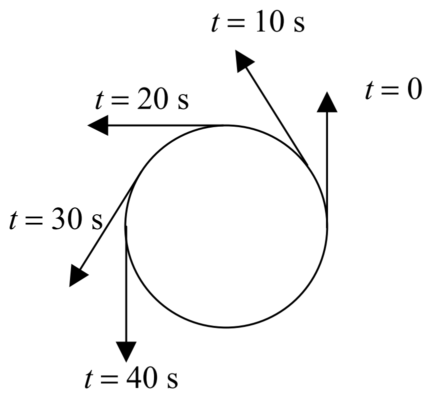
$\mathbf{v} = \underline{-(21\ \text{km/h})\mathbf{i} + (21\ \text{km/h})\mathbf{j}}$. At $t = 20$ s,

$\mathbf{v}$ = 30 km/h W, or $\mathbf{v} = \underline{-(30\ \text{km/h})\mathbf{i} + 0\mathbf{j}}$. At $t = 30$ s,

$\mathbf{v}$ = 30 km/h @ 45° S of W, or

$\mathbf{v} = \underline{-(21\ \text{km/h})\mathbf{i} - (21\ \text{km/h})\mathbf{j}}$. At $t = 40$ s, the driver is halfway around and $\mathbf{v}$ = 30 km/h S, or

$\mathbf{v} = \underline{(30\ \text{km/h})\mathbf{i} + 0\mathbf{j}}$.



4-5. $\mathbf{r} = (4 + 2t)\mathbf{i} + (3 + 5t + 4t^2)\mathbf{j} + (2 - 2t - 3t^2)\mathbf{k}$

(a) $\mathbf{v} = d\mathbf{r}/dt = \underline{2\mathbf{i} + (5 + 8t)\mathbf{j} - (2 + 6t)\mathbf{k}}$

(b) $\mathbf{a} = d\mathbf{v}/dt = \underline{8\mathbf{j} - 6\mathbf{k}}$

magnitude, $|\,\mathbf{a}\,| = \sqrt{(8)^2 + (6)^2} = \underline{10\ \text{m/s}^2}$

direction $\theta = -\tan^{-1}\left(\dfrac{6}{8}\right) = \underline{-37°}$ (37° below the y-axis in the z-y plane)

4-6. (a) The components of **v** are $v_x = dx/dt$ and $v_y = dy/dt$. Also, the components of **a** are $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

$v_x = \underline{-Ab\sin bt}$ $\qquad v_y = \underline{Ab\cos bt}$

$a_x = \underline{-Ab^2\cos bt}$ $\qquad a_y = \underline{-Ab^2\sin bt}$

(b) $|\mathbf{v}| = \sqrt{(-Ab\sin bt)^2 + (ab\cos bt)^2} = \underline{\sqrt{A^2b^2(\sin^2 bt + \cos^2 bt} = Ab}$

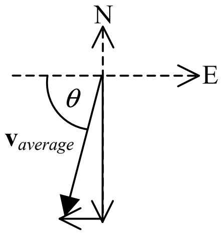
$|\mathbf{a}| = \sqrt{(-Ab^2\cos bt)^2 + (-Ab^2\sin bt)^2} = \underline{Ab^2\sqrt{\cos^2 bt + \sin^2 bt} = Ab^2}$

†4-7. At $t = 2.0$ s, the missile has been falling with acceleration $\mathbf{a} = 0\mathbf{i} - (9.81\ \text{m/s}^2)\mathbf{j}$ and horizontal velocity equal to the velocity of the airplane. This means the missile is still directly below the airplane. Its displacement relative to the plane is $\mathbf{r} = 0\mathbf{i} - \frac{gt^2}{2}\mathbf{j} = 0\mathbf{i} - \frac{(9.81\ \text{m/s}^2)(2.0\ \text{s})^2}{2}\mathbf{j} =$ $0\mathbf{i} - (19.6\ \text{m})\mathbf{j}$, or 19.6 m @ 90º below the direction of travel of the airplane. At $t = 3.0$ s, which is 1.0 s after igniting the engine, the acceleration is $\mathbf{a} = (6.0\ \text{m/s}^2)\mathbf{i} - (9.81\ \text{m/s}^2)\mathbf{j}$. Its displacement during this 1.0 s interval is $\mathbf{r}_1 = \frac{a_x t^2}{2}\mathbf{i} - \frac{gt^2}{2}\mathbf{j}$ $= \frac{(6\ \text{m/s}^2)t^2}{2}\mathbf{i} - \frac{gt^2}{2}\mathbf{j} = \frac{(6\ \text{m/s}^2)(1.0\ \text{s})^2}{2}\mathbf{i} - \frac{(9.81\ \text{m/s}^2)(1.0\ \text{s})^2}{2}\mathbf{j} = (3.0\ \text{m})\mathbf{i} - (4.9\ \text{m})\mathbf{j}$. Now the missile's total displacement relative to the plane is $\mathbf{r}_2 = (0\ \text{m})\mathbf{i} - (19.6\ \text{m})\mathbf{j} + (3.0\ \text{m})\mathbf{i} - (4.9\ \text{m})\mathbf{j} = (3.0\ \text{m})\mathbf{i} - (24.5\ \text{m})\mathbf{j}$. The magnitude is $r_2 = \sqrt{(3.0\ \text{m})^2 + (24.5\ \text{m})^2} = 24.7\ \text{m}$. The direction is given by $\theta = \tan^{-1}\frac{-24.5}{3} = -83.0°$, or 24.7 m @ 83º below the direction of travel of the plane.

4-8. $\mathbf{r} = (5t + 4t^2)\mathbf{i} + (3t^2 + 2t^3)\mathbf{j} + 0\mathbf{k}$ m. $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \underline{(5 + 8t)\mathbf{i} + (6t + 6t^2)\mathbf{j} + 0\mathbf{k}\ \text{m/s}}$.

$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} = \frac{d\mathbf{v}}{dt} = (8)\mathbf{i} + (6 + 12t)\mathbf{j} + 0\mathbf{k}\ \text{m/s}^2$. At $t = 2.0$ s, $\mathbf{v} = \underline{(21\ \text{m/s})\mathbf{i} + (36\ \text{m/s})\mathbf{j} + 0\mathbf{k}\ \text{m/s}}$, which gives a speed of $v = \sqrt{(21\ \text{m/s})^2 + (36\ \text{m/s})^2} = \underline{42\ \text{m/s}}$.

†4-9. $\mathbf{v}_{average} = \frac{\Delta\mathbf{r}}{\Delta t}$, where $\Delta\mathbf{r}$ is the total displacement and $\Delta t = 1.5$ h is the total elapsed time. To find the total displacement, find the displacement during each part of the trip:
$\Delta\mathbf{r}_1 = (300\ \text{km/h @ }30°\ \text{N of E}) \times 0.50\ \text{h} = 150\ \text{km @ }30°\ \text{N of E}$. Taking the y direction to point N and the x direction to point E, this is $\Delta\mathbf{r}_1 = (150\ \text{km})(\cos\ 30°)\mathbf{i} + (150\ \text{km})(\sin 30°)\mathbf{j}$ $= (130\ \text{km})\mathbf{i} + (75\ \text{km})\mathbf{j}$. For the second part of the trip,
$\Delta\mathbf{r}_2 = (300\ \text{km/h @ }30°\ \text{W of S}) \times 1.0\ \text{h} = 300\ \text{km @ }30°\ \text{W of S}$. In terms of x and y, this is $\Delta\mathbf{r}_2 = -(300\ \text{km})(\sin\ 30°)\mathbf{i} - (300\ \text{km})(\cos 30°)\mathbf{j} = -(150\ \text{km})\mathbf{i} - (260\ \text{km})\mathbf{j}$. The total displacement is $\Delta\mathbf{r} = \Delta\mathbf{r}_1 + \Delta\mathbf{r}_2 = -20\ \text{km}\ \mathbf{i} - 185\ \text{km}\ \mathbf{j}$. The average velocity is $\mathbf{v}_{average} = \frac{\Delta\mathbf{r}}{\Delta t}$

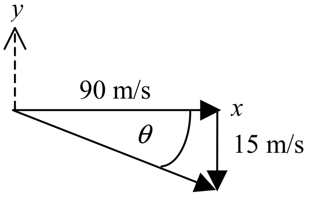


$= \dfrac{-20\text{ km}\ \ \mathbf{i} - 185\text{ km}\ \mathbf{j}}{1.5\text{ h}}$ $= -13.3\text{ km/h}\ \mathbf{i} - 123\text{ km/h}\ \mathbf{j}$. To give this as a speed and heading, use $v_{average} = \sqrt{13.3^2 + 123^2}\text{km/h} = 124\text{ km/h}$. Since both velocity components are negative, the vector is located in the third quadrant (between W and S), so we can give the direction as $\theta = \tan^{-1}\dfrac{123}{13.3}$ = 83.8° S of W. Thus $\mathbf{v}_{average}$ = 124 km/h @ 83.8° S of W.

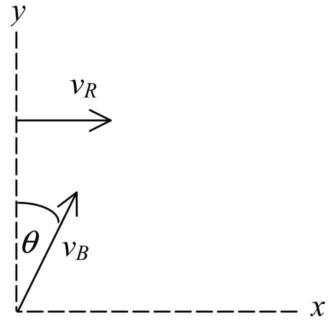
The average acceleration is $\mathbf{a}_{average} = \dfrac{\Delta\mathbf{v}}{\Delta t} = \dfrac{\mathbf{v}_2 - \mathbf{v}_1}{\Delta t}$, where $\mathbf{v}_1$ is the velocity during the 0.5, $\mathbf{v}_2$ is the velocity during the next 1.0 h, and Δt is the time interval during which the velocity changes. Since no value is given for Δt, we can calculate $\Delta\mathbf{v}$ and give the answer symbolically. Using the definitions above, $\mathbf{v}_1 = (300\text{ km/h})\cos 30°\mathbf{i} + (300\text{ km/h})\sin 30°\mathbf{j}$ $= (260\text{ km/h})\mathbf{i} + (150\text{ km/h})\mathbf{j}$. 30° W of S is the same as 240° N of E, so $\mathbf{v}_2 = (300\text{ km/h})\cos 240°\mathbf{i} + (300\text{ km/h})\sin 240°\mathbf{j}$ $= -(150\text{ km/h})\mathbf{i} - (240\text{ km/h})\mathbf{j}$. Then $\Delta\mathbf{v} = -\,(410\text{ km/h})\mathbf{i} - (410\text{ km/h})\mathbf{j}$, and $\mathbf{a}_{average} = \dfrac{-(410\text{ km/h})\mathbf{i} - (410\text{ km/h})\mathbf{j}}{\Delta t}$. As a magnitude and heading, this is $\mathbf{a}_{average} = \dfrac{410\sqrt{2}\text{ km/h}}{\Delta t}$ @ 45° S of W, or $\mathbf{a}_{average} = \dfrac{580\text{ km/h}}{\Delta t}$ @ 45° S of W. Even though we don't have a numerical value, it is important to note that the average acceleration is *not* zero, even though the speed of the airplane is constant. The acceleration is caused by the change in direction of the velocity.

4-10. $\mathbf{r} = A\cos bt\mathbf{i} + Bt\mathbf{j}$. $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = -\,bA\sin bt\mathbf{i} + B\mathbf{j}$. $\mathbf{a} = \dfrac{d^2\mathbf{r}}{dt^2} = \dfrac{d\mathbf{v}}{dt} = -\,b^2A\cos bt\mathbf{i} + 0\mathbf{j}$. The equation for the speed is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(bA\sin bt)^2 + B^2}$.

†4-11. $\mathbf{r} = 90t\mathbf{i} + (500 - 15t)\mathbf{j}\text{ m}$. $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 90\mathbf{i} - 15\mathbf{j}\text{ m/s}$. The equation for the speed is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(90\text{ m/s})^2 + (15\text{ m/s})^2} = 91\text{ m/s}$. The direction of the velocity is $\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\left(\dfrac{-15}{90}\right) = -\,9.5°$. This is 9.5° below the x axis.



4-12. Measure time from when the ball is thrown. At that instant, the receiver has traveled (7 m/s)(2.0 s) = 14 m along the x direction relative to the quarterback. The receiver is at an initial displacement $\mathbf{r}_0 = (14 \text{ m})\mathbf{i} + (20 \text{ m})\mathbf{j}$ relative to the quarterback. Since he's running at a constant velocity of $\mathbf{v}_R = 7.0\mathbf{i}$ m/s, his total displacement is $\mathbf{r}_R = (14 + 7t)\mathbf{i} + 20\,\mathbf{j}$ m. Once it's thrown, the displacement of the ball relative to the quarterback is $\mathbf{r}_B = v_{B,x}t\mathbf{i} + v_{B,y}t\mathbf{j} = (v_B \sin\theta)t\mathbf{i} + (v_B \cos\theta)t\mathbf{j}$, where $v_B = 15$ m/s. Mathematically, the ball is caught when $\mathbf{r}_B = \mathbf{r}_R : \sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$. Since the x components must be equal and the y components must be equal, we get two simultaneous equations:



$$(15\sin\theta)t = 14 + 7t$$

$$(15\cos\theta)t = 20$$

Use the second equation to get $t = \frac{20}{15\cos\theta} = \frac{4}{3\cos\theta}$. Substitute this into the first equation to get $15\sin\theta\left(\frac{4}{3\cos\theta}\right) = 14 + 7\left(\frac{4}{3\cos\theta}\right)$. Simplifying gives $\underline{30\sin\theta - 21\cos\theta = 14.}$

This transcendental equation can be solved by trial and error, by graphical means, or by use of tools like Solver in Excel or a Solve Block in MathCAD. The result is $\theta = \underline{57.6°}$. The flight time for the ball is $\underline{t = \frac{4}{3\cos 57.6°} = 2.5 \text{ s.}}$

†4-13. $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3\mathbf{i} + 2\mathbf{j} \Rightarrow d\mathbf{v} = (3\mathbf{i} + 2\mathbf{j})dt$. Integrate to find $\mathbf{v}$: $\int_{\mathbf{v}_0}^{\mathbf{v}} d\mathbf{v} = \int_0^t (3\mathbf{i} + 2\mathbf{j})dt$, which gives $\mathbf{v} - \mathbf{v}_0 = 3t\mathbf{i} + 2t\mathbf{j}$. The problem states that both components of velocity are initially zero, so we get $\mathbf{v} = \underline{3t\mathbf{i} + 2t\mathbf{j} \text{ m/s}}$. Since $\mathbf{v} = \frac{d\mathbf{r}}{dt}$, we can follow the same procedure that was used to get $\mathbf{v}$ from $\mathbf{a}$: $d\mathbf{r} = (3t\mathbf{i} + 2t\mathbf{j})dt \Rightarrow \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r} = \int_0^t (3t\mathbf{i} + 2t\mathbf{j})dt \Rightarrow \mathbf{r} - \mathbf{r}_0 = \frac{3t^2}{2}\mathbf{i} + t^2\mathbf{j}$. The problem says the particle starts moving at the origin, so both components of $\mathbf{r}_0$ are zero. $\mathbf{r} = \underline{\frac{3t^2}{2}\mathbf{i} + t^2\mathbf{j} \text{ m.}}$

4-14. 162.3 km/h = 45.1 m/s

Time to travel 20m: $t = \frac{20}{4501} = 0.44$ s

The distance fallen vertically: $z = \frac{1}{2}gt^2 = \frac{1}{2}(9.8)(0.44)^2 = \underline{0.95 \text{ m}}$

†4-15. Assume the divers launch themselves horizontally from the edge of the cliff, so $v_{oy} = 0$. The time to fall a distance $h = 64$ m is $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(36\text{ m})}{9.81\text{ m/s}^2}} = 2.71$ s. During this time, the diver must travel a horizontal distance of at least $x = 6.4$ m, so the minimum horizontal velocity required is $v_{0x} = \dfrac{x}{t} = \dfrac{6.4\text{ m}}{2.71\text{ s}} = 2.4\text{ m/s}$.

4-16. From Example 6, $t = 3.19$ s (An extra digit has been kept so the final results of this problem can safely be rounded to the two digits required by the data given in the Example.)

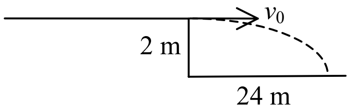
(a) $v_x = 83.3\text{ m/s}$ (83 m/s to two digits) ($a_x = 0$, so the horizontal velocity is constant)

$v_y = -gt = -(9.81\text{ m/s}^2)(3.19\text{ s}) = -31.3\text{ m/s}$. To two digits, $v_y = -31\text{ m/s}$.

(b) $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(83.3\text{ m/s})^2 + (31.3\text{ m/s})^2} = 89\text{ m/s}$. The speed has increased by about 6 m/s.

†4-17. The stunt car will fall 2 m while it travels 24 m horizontally.

The time to fall 2 m is $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(2\text{ m})}{9.81\text{ m/s}^2}} = 0.639$ s. The car must travel 24 m horizontally in 0.639 s, so its horizontal speed must be 38 m/s. (This is about 135 kph, or 84 mph.)



4-18. $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t = (3\mathbf{i} + 2\mathbf{j}) + (\mathbf{i} - 4\mathbf{j})t = (3 + t)\,\mathbf{i} + (2 - 4t)\,\mathbf{j}$

The maximum y-coordinate is reached when the y-component of the velocity is zero. Thus, $2 - 4t = 0$ or $t = 0.5$ s

$\mathbf{r} = \mathrm{r}_o^0 + \mathbf{v}_0 t + \dfrac{1}{2}\mathbf{a}\,t^2 = 3t\,\mathbf{i} + 2t\,\mathbf{j} + 0.5t^2\,\mathbf{i} - 2t^2\,\mathbf{j} = (3t + 0.5t^2)\,\mathbf{i} + (2t - 2t^2)\,\mathbf{j}$.

At $t = 0.5$ s, $\mathbf{r} = (1.625\,\mathbf{i} + 0.5\,\mathbf{j})$ m

4-19. The maximum distance, $x_{\max} = v_0^2/g$

(i) $v_0 = \sqrt{x_{\max} g} = \sqrt{441 \times 9.8} = 65.7\text{ m/s}$

(ii) $v_0 = \sqrt{889 \times 9.8} = 93.3\text{ m/s (336 km/h)}$

4-20. $x_{\max} = v_0^2 \sin 2\theta/g$ with $\theta = 12°$, $x_{\max} = 250$ m

$$v_0^2 = \frac{g x_{\max}}{\sin 2\theta} = \left(\frac{9.81 \times 250}{\sin 24°}\right)\text{m}^2/\text{s}^2 = 6030\text{ m}^2/\text{s}^2$$

$v_o = 77.7\text{ m/s}$

$$\text{max ht} = z_{\max} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{6030 \sin^2 12°}{2 \times 9.81}\text{ m} = 13.3\text{ m}$$

4-21. (a) The horizontal range is given by $x_{target} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g}$. Using $\sin 2\theta = 2\sin\theta\cos\theta$ gives

$$\sin 2\theta = \frac{g x_{target}}{v_0^2}.\ \text{Thus}\ \theta = \frac{1}{2}\sin^{-1}\left(\frac{g x_{target}}{v_0^2}\right),\ \text{so}\ \theta = \frac{1}{2}\sin^{-1}\left[\frac{(9.81\text{ m/s}^2)(12{,}500\text{ m})}{(700\text{ m/s})^2}\right] = 7.25°.$$

(b) In this part of the problem, it will be necessary to assume that all the digits given are significant, and that the values of g are specified exactly as given. Suppose the gunner uses g = 9.80 m/s^2 in the calculations. Then the elevation angle would be calculated using $x_{target} = \dfrac{v_0^2 \sin 2\theta^*}{9.80\ \text{m/s}^2}$, where x_{target} is the given distance to the target and θ^* is the *wrong* angle (associated with g = 9.80 m/s^2). As in part (a), $\sin 2\theta^* = \dfrac{(9.80\ \text{m/s}^2)x_{t\,arget}}{\text{v}_0^2}$. But the actual range will be $x_{max} = \dfrac{v_0^2 \sin 2\theta^*}{9.81\ \text{m/s}^2}$. Substituting the expression for $\sin 2\theta^*$ gives $x_{max} = x_{target}\left(\dfrac{9.80}{9.81}\right)$ $= (12{,}500\ \text{m})\left(\dfrac{9.80}{9.81}\right) = \underline{12{,}487\ \text{m}}$. The gunner will miss the target by $\underline{13\ \text{m}}$.

4-22. Initial speed, $v_0 = \sqrt{x_{\text{max}}\, g} = \sqrt{1500 \times 9.8} = \underline{121.2\ \text{m/s}}$

$$z_{\text{max}} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{x_{\text{max}}}{4} = \underline{375\ \text{m}}$$

$$t_{\text{flight}} = \frac{2v_0 \sin\theta}{g} = \frac{2 \times 121.2}{\sqrt{2} \times 9.8} = \underline{17.5\ \text{s}}$$

4-23. (a) $v_{\text{ty}} = 0 = v_{\text{iy}} - gt$

$$t = \frac{v_{\text{iy}}}{g} = \left(\frac{87 \times 10^3}{60 \times 60}\right)\frac{\sin 45}{9.8} = \underline{1.74\ \text{s}}$$

(b) $z_{\text{max}} = \dfrac{v_{iy}^2}{2g} = \left(\dfrac{87 \sin 45}{3.6}\right)^2 \dfrac{1}{2 \times 9.8} = \underline{14.9\ \text{m}}$

(c) $x_{\text{max}} = \dfrac{v_0^2}{g} = \left(\dfrac{87}{3.6}\right)^2 \dfrac{1}{9.8} = \underline{59.6\ \text{m}}$

4-24. Denote Berlin by x_1, g_1 and Buenos Aires by x_2, g_2.

$$x_1 = \frac{v_0^2 \sin 2\theta}{g_1} \qquad x_2 = \frac{v_0^2 \sin 2\theta}{g_2}$$

Then $\dfrac{x_1}{x_2} = \dfrac{g_2}{g_1} \geq x_2 = \dfrac{x_1 g_1}{g_2} = 68.11\ \text{m} \times \dfrac{9.8128}{9.7967} = \underline{69.22\ \text{m}}$

$\Delta x = \underline{0.11\ \text{m}}$

4-25. $v_0 = \sqrt{x_{\text{max}}\ \text{s}} = \sqrt{10^6 \times 9.8} = \underline{3.13 \times 10^3\ \text{m/s}}$

$$z_{\text{max}} = \frac{v_{\text{y}}^2}{2g} = \frac{(3.13 \times 10^3)^2 \sin^2 45}{2 \times 9.8} = \underline{2.5 \times 10^5\ \text{m}}$$

$$t_{\text{flight}} = \frac{2v_0 \sin 45^\circ}{g} = \frac{2 \times 3.13 \times 10^3 \sin 45^\circ}{9.8} = \underline{452\ \text{s}}$$

4-26. (a) $x_{\max} = v_0^2/g \geq v_0 = \sqrt{x_{\max}\, g} = \sqrt{50 \times 9.8}$ m/s $= \underline{22 \text{ m/s}}$

(b) The circumference of the circle is $2\pi \times 1.0$ m = 6.28 m.

$$t_{\text{rev}} = \frac{\text{dist}}{\text{speed}} = \frac{6.28 \text{ m}}{22 \text{ m/s}} = 0.285 \text{ s}$$

Thus, number of rev/s = $\dfrac{1}{t_{\text{rev}}} = \dfrac{1}{0.285}$ s $= \underline{3.5 \text{ rev/s}}$

4-27. $$x_{\max} = \frac{v_0^2 \sin 2\theta}{g} = \frac{(26)^2 \sin 70°}{9.8} = \underline{64.8 \text{ m}}$$

$$t_{\text{flight}} = \frac{2v_0 \sin\theta}{g} = \frac{2\,(26)\,\sin 35°}{9.8} = \underline{3.04 \text{ s}}$$

The amount of water in the air at any given instant is $\dfrac{280}{60} \times 3.04 = \underline{14.2 \text{ liters}}$

4-28. $x_{\max} = v_0^2/g \geq v_0 = \sqrt{x_{\max}\, \text{g}} = \sqrt{730 \times 9.8}$ m/s $= \underline{85 \text{ m/s}}$

$$t_{\text{flight}} = \frac{2v_0 \sin\theta}{g} = \frac{2 \times 85 \times \sin 45°}{9.8} \text{ s} = \underline{12 \text{ s}}$$

4-29. $x_{max} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g}$, $y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g}$. $x_{max} = y_{max} \Rightarrow \dfrac{2v_0^2 \sin\theta\cos\theta}{g} = \dfrac{v_0^2 \sin^2\theta}{2g}$. This gives $2\cos\theta = \dfrac{\sin\theta}{2}$, or $\tan\theta = 4 \Rightarrow \theta = \underline{76°}$.

4-30. $v_{0y} = \sqrt{2gy_{max}} = \sqrt{2(9.81 \text{ m/s}^2)(0.60 \text{ m})} = 3.43$ m/s. $t_{flight} = \dfrac{2v_{0y}}{g} = \dfrac{2(3.43 \text{ m/s})}{9.81 \text{ m/s}^2} = \underline{0.70 \text{ s}}$. (The exact angle at which the balls are thrown is not relevant.)

†4-31. $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = 2.0\mathbf{i} - 4.5\mathbf{j} \Rightarrow d\mathbf{v} = (2.0\mathbf{i} - 4.5\mathbf{j})dt$. Integrate to find **v** following the method used in Problem 4-13: $\mathbf{v} - \mathbf{v}_0 = \int_0^t (2.0\mathbf{i} - 4.5\mathbf{j})dt = 2.0t\mathbf{i} - 4.5t\mathbf{j}$. The problem states that $\mathbf{v}_0 = -10\mathbf{i} + 25\mathbf{j}$, so we get $\mathbf{v} = 2.0t\mathbf{i} - 4.5t\mathbf{j} + (-10\mathbf{i} + 25)\mathbf{j} = (2.0t - 10)\mathbf{i} + (25 - 4.5t)\mathbf{j}$ m/s.

At $t = 3.0$ s, the velocity is $\mathbf{v} = (6.0 - 10)\mathbf{i} + (25 - 13.5)\mathbf{j} = -4.0\mathbf{i} + 11.5\mathbf{j}$ m/s. The speed is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(4.0 \text{ m/s})^2 + (11.5 \text{ m/s})^2} = \underline{12 \text{ m/s}}$.

Since $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$, we can follow the same procedure that was used to get **v** from **a**:

$$d\mathbf{r} = [(2.0t - 10)\mathbf{i} + (25 - 4.5t)\mathbf{j}]dt \Rightarrow \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r} = \int_0^t [(2.0t - 10)\mathbf{i} + (25 - 4.5t)\mathbf{j}]dt \Rightarrow$$

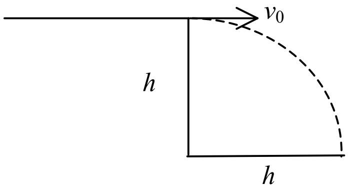
$\mathbf{r} - \mathbf{r}_0 = (t^2 - 10t)\mathbf{i} + (25t - 2.25t^2)\mathbf{j}$ m. The problem says the particle starts moving at the origin, so both components of $\mathbf{r}_0$ are zero. $\mathbf{r} = \underline{(t^2 - 10t)\mathbf{i} + (25t - 2.25t^2)\mathbf{j} \text{ m}}$. At $t = 3.0$ s, $\mathbf{r} = \underline{-21\mathbf{i} + 55\mathbf{j} \text{ m}}$.

4-32. $t_{flight} = \frac{2v_0 \sin\theta}{g} \Rightarrow v_0 = \frac{gt_{flight}}{2\sin\theta}$. $x_{max} = \frac{2v_0^2 \sin\theta\cos\theta}{g} = \frac{2\sin\theta\cos\theta}{g}\left(\frac{gt_{flight}}{2\sin\theta}\right)^2 = \frac{gt^2_{flight}}{2\tan\theta} \Rightarrow$

$\theta = \tan^{-1}\left(\frac{gt^2_{flight}}{2x_{max}}\right)$. $\theta = \tan^{-1}\left[\frac{(9.81\text{ m/s}^2)(6.0\text{ s})^2}{2(75\text{ m})}\right] = \underline{67°}$.

4-33. $x_{max} = \frac{2v_0^2 \sin\theta\cos\theta}{g} \Rightarrow v_0 = \sqrt{\frac{gx_{max}}{2\sin\theta\cos\theta}} = \sqrt{\frac{(9.81\text{ m/s}^2)(15\text{ m})}{2\sin 10°\cos 10°}} = \underline{21\text{ m/s}}$.

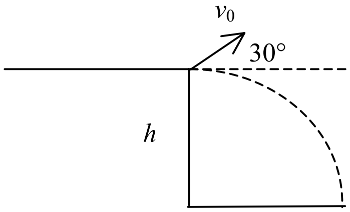
4-34. The time to fall a distance h is $t = \sqrt{\frac{2h}{g}}$. The object must also travel the same horizontal distance h during this time, so $v_0 = \frac{h}{t} = \sqrt{\frac{gh}{2}}$. After time t the vertical velocity will be $v_y = -gt = -\sqrt{2gh}$, so the final velocity at time of impact is



$\underline{\mathbf{v} = \sqrt{\frac{gh}{2}}\mathbf{i} - \sqrt{2gh}\mathbf{j}}$.

†4-35. The time to reach maximum height is $t_{height} = \frac{2.25\text{ s}}{2} = 1.125\text{ s}$. $t_{height} = \frac{v_{0y}}{g} \Rightarrow$ $v_{0y} = gt_{height} = (9.81\text{ m/s}^2)(1.125\text{ s}) = 11.0\text{ m/s}$. If we say the launch point was at $y = 0$, then the equation for the position below the launch point is $y = v_{0y}t - \frac{gt^2}{2}$ with $v_{0y} = 11.0\text{ m/s}$. This gives $y = (11.0\text{ m/s})(4.00\text{ s}) - \frac{(9.81\text{ m/s}^2)(4.00\text{ s})^2}{2} = -34.5\text{ m}$. The cliff is $\underline{34.5\text{ m}}$ high. Since the launch angle was 45°, $v_{0x} = v_{0y} = 11.0\text{ m/s}$. The total horizontal distance traveled since launch is $x = v_{0x}t_{total} = (11.0\text{ m/s})(2.25\text{ s} + 4.00\text{ s}) = \underline{68.8\text{ m}}$.

4-36. Choose the origin of the coordinate system to be at the bottom of the cliff. Then $y = y_0 + v_{0y}t - \frac{gt^2}{2}$. $y_0 = h = 30\text{ m}$; $v_0 = 25\text{ m/s}$. $v_{0y} = v_0 \sin 30° = 12.5\text{ m/s}$. At the bottom, $y = 0$, so we must solve this quadratic equation: $0 = 30 + 12.5t - 4.91t^2$. The roots are given by $t = \frac{-12.5 \pm \sqrt{(12.5)^2 + 4(4.91)(30)}}{-9.81} = -1.51\text{ s}, 4.06\text{ s}$. The final result must be positive, so $t = \underline{4.06\text{ s}}$.



4-37. The flight time to the goal is $t = \frac{x}{v_0\cos\theta} = \frac{25\text{ m}}{(65\text{ m/s})\cos 10°} = \underline{0.391\text{ s}}$. The height of the puck at this time is $y = v_0 \sin\theta t - \frac{gt^2}{2} = (65\text{ m/s})(\sin 10°)(0.391\text{ s}) - \frac{(9.81\text{ m/s}^2)(0.391\text{ s})^2}{2} = \underline{3.66\text{ m}}$. The puck will pass 2.2 m above the goal.

4-38. (a) $x_{max} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g} \Rightarrow v_0 = \sqrt{\dfrac{gx_{max}}{2\sin\theta\cos\theta}} = \sqrt{\dfrac{(9.81\text{ m/s}^2)(240\text{ m})}{2(\sin 14°)(\cos 14°)}} = \underline{70.8\text{ m/s}}$.

(b) $x_{max} = \dfrac{2(71.4\text{ m/s})^2 \sin 14°\cos 14°}{9.81\text{ m/s}^2} = \underline{244\text{ m}}$. The ball will overshoot by about $\underline{4\text{ m}}$.

(c) $x_{max} = \dfrac{2(70.8\text{ m/s})^2 \sin 14.5°\cos 14.5°}{9.81\text{ m/s}^2} = \underline{248\text{ m}}$. The ball will overshoot by about $\underline{8\text{ m}}$.

4-39. $x_{max} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g}$, $y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g}$. For $\theta = 45°$, $\sin\theta = \cos\theta = \dfrac{1}{\sqrt{2}}$. Substituting in the equations for x_{max} and y_{max} gives $x_{max} = \dfrac{2v_0^2}{2g} = \dfrac{v_0^2}{g}$, $y_{max} = \dfrac{v_0^2}{4g}$. Taking the ratio gives $\underline{\dfrac{y_{max}}{x_{max}} = \dfrac{1}{4}}$, which is the desired result.

4-40. $z_{\max} = \dfrac{v_0^2 \sin^2\theta}{2g} = 8.667\text{ ft}$ (i)

$x_{\max} = \dfrac{v_0^2 \sin 2\theta}{g} = 37\text{ ft}$ (ii)

Dividing (i) by (ii) gives $\dfrac{\sin^2\theta}{\sin 2\theta} = 0.2342$

$\dfrac{\sin^2\theta}{\sin 2\theta} = \dfrac{\sin^2\theta}{4\sin\theta\cos\theta} = \dfrac{1}{4}\tan\theta = 0.2342$

$\theta = \tan^{-1}(4 \times 0.2342) = \underline{43.1°}$

From (ii) $v_0^2 = 37 \times 32.2/\sin 86.2° = 1194\text{ ft}^2/\text{s}^2$

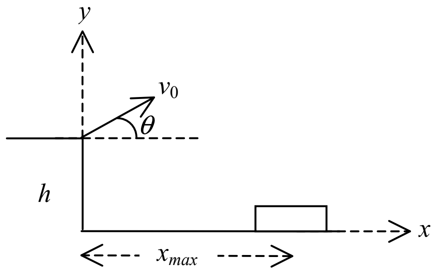
$v_0 = \underline{34.6\text{ ft/s}}$

†4-41. $x_{max} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g}$, $y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g}$. Setting these equal to each other gives $\dfrac{2v_0^2 \sin\theta\cos\theta}{g} = \dfrac{v_0^2 \sin^2\theta}{2g}$, from which we get $\dfrac{\sin\theta}{\cos\theta} = \tan\theta = 2$, or $\theta = \tan^{-1} 2 = \underline{63.4°}$.

4-42. $t_{\text{flight}} = \dfrac{2v_0 \sin\theta}{g} = \dfrac{2 \times 38 \sin 52°}{9.8} = 6.1\text{ s}$

In order to catch the ball, the fielder must run with a minimum speed of 45/6.1 = 7.4 m/s. Therefore with 8 m/s the fielder will be able to catch the ball.

†4-43. A rough sketch (not to scale) is shown. The coordinates of the projectile are



$y = h + v_0(\sin\theta)t - \dfrac{gt^2}{2}$

$x = v_0(\cos\theta)t$

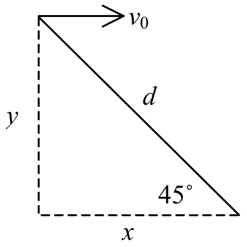
The coordinates of the target are (x_{max},0). The flight time then is $t = \dfrac{x_{max}}{v_0\cos\theta}$. Set $y = 0$ and substitute this time.

After some rearranging, the equation becomes $g\left(\dfrac{x_{max}^2}{v_0^2\cos^2\theta}\right) - 2v_0\sin\theta\left(\dfrac{x_{max}}{v_0\cos\theta}\right) - 2h = 0.$ We can convert this to a quadratic equation with tan θ as the variable by using the following trigonometric steps: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}.\ \tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1-\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta} - 1.$ Thus $\dfrac{1}{\cos^2\theta} = 1 + \tan^2\theta,$ and we have $\left(\dfrac{gx_{max}^2}{v_0^2}\right)(1+\tan^2\theta) - 2x_{max}\tan\theta - 2h = 0.$ Multiplying by $\left(\dfrac{v_0^2}{gx_{max}^2}\right)$ and rearranging finally gives this equation:

$\tan^2\theta - 2x_{max}\left(\dfrac{v_0^2}{gx_{max}^2}\right)\tan\theta + 1 - 2h\left(\dfrac{v_0^2}{gx_{max}^2}\right) = 0.$ Using $x_{max} = 12 \times 10^3$ m, $v_0 = 600$ m/s, and $g =$ 9.81 m/s^2 gives $\left(\dfrac{v_0^2}{gx_{max}^2}\right) = 2.548 \times 10^{-4}$ m^{-1}. Substituting $h = 50$ m and doing the rest of the arithmetic finally gives $\underline{\tan^2\theta - 6.116\tan\theta + 0.9745 = 0}.$ The solution is

$\tan\theta = \dfrac{6.116 \pm \sqrt{6.116^2 - 4(0.9745)}}{2} = 5.953,\ 0.1635.$ The two angles are the inverse tangents of these: $\theta = \underline{80.5°, 9.29°}$. It turns out that most projectile aiming problems like this have two solutions. The difference between the two angles is that the larger elevation will give a longer flight time and the projectile will strike the target at a larger angle than for the smaller elevation.

4-44. (a) $y = -\dfrac{gt^2}{2},\ x = v_0 t.$ The distance d down the slope is related to x and y by the slope angle of 45°: $x = d\cos 45°,\ y = d\sin 45°.$ Substitute these into the equations for x and y. Solve the x equation for t: $t = \dfrac{d\ \cos\ 45°}{v_0}$, Substitute into the y equation:



$d\sin 45° = \dfrac{g}{2}\left(\dfrac{d\cos 45°}{v_0}\right)^2$. Solve for d:

$d = \dfrac{2v_0^2\sin 45°}{g\cos^2 45°}.\ \dfrac{2\ \sin 45°}{\cos^2 45°} = \sqrt{2},$ so $d = \dfrac{\sqrt{2}v_0^2}{g}.$ $v_0 = 110$ km/h $= 30.6$ m/s

$\Rightarrow d = \dfrac{\sqrt{2}(30.6\text{ m/s})^2}{9.81\text{ m/s}^2} = \underline{135\text{ m}}.$

(b) Skiers are able to land farther down the slope by arching their bodies forward over the skis. This moves their center of gravity forward, reduces aerodynamic drag, and actually provides some aerodynamic lift which carries them farther down the slope than would be predicted by projectile motion analysis.

4-45. $x = 90 = (70\cos\theta)\,t$ (i)

$y = 0 = (70\sin\theta)\,t - \dfrac{1}{2}gt^2$ (ii)

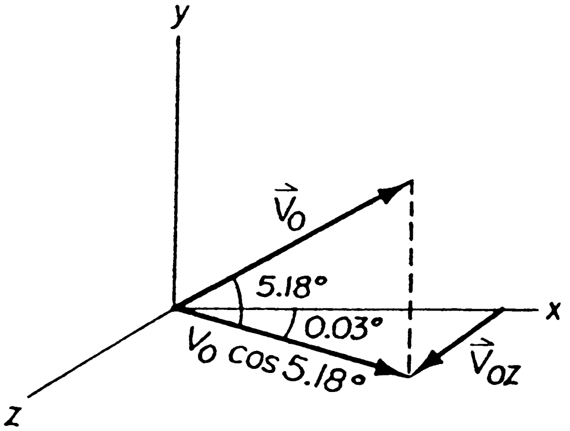
Substitute for t from (ii) in (i) to get $x = \dfrac{v_0^2\ \sin 2\theta}{g}$

$$\theta = \frac{1}{2}\,\sin^{-1}\left(\frac{x\,g}{v_0^2}\right) = \frac{1}{2}\,\sin^{-1}\left(\frac{90\times 9.8}{(70)^2}\right) = 5.18°$$

when the arrow is misaimed by 0.03° in the vertical direction,

$$y = x\tan\theta - \frac{1}{2}\,g\left(\frac{90}{70}\right)^2\frac{1}{\cos^2\theta} = 90\tan(5.21°) - \frac{1}{2}9.8\left(\frac{9}{7}\right)^2\frac{1}{\cos^2 5.21°}$$

$= -3.9\times 10^{-2}$ m $=$ −3.9 cm (Arrow hits bull's-eye)



A misaim of 0.03° in the horizontal direction will give the arrow a component of velocity in the z-direction

$v_{0z} = v_0 \cos 5.18° \sin 0.03$

The deviation in the horizontal direction from the center of the bull's-eye is

$$v_{0z}t = (70\cos 5.18°\sin 0.03°)\,\frac{90}{70\cos 5.18°}$$

$= 4.7\times 10^{-2}$ m $=$ 4.7 cm

Therefore the arrow will hit the bull's-eye.

4-46. (a) $x_{max} = \dfrac{v_0^2\sin 2\theta}{g} \Rightarrow \sin 2\theta = \sqrt{\dfrac{x_{max}\,g}{v_0^2}}$

$\sin 2\,\theta = \sqrt{(700\text{ m}\times 9.8\text{ m/s}^2)/(630\text{ m/s})^2} = 0.01728$

$\theta = \dfrac{1}{2}\sin^{-1}(0.01728) = \underline{0.50°}$

If **d** is the distance above the target, then $\tan\theta = d/700$ or $d = 700$ m tan 0.5°

$d = \underline{6\text{ m}}$

(b) $z_{max} = \dfrac{v_0^2\sin^2\theta}{2g} = \dfrac{630^2\sin^2(0.5)}{2\times 9.8} = \underline{1.5\text{ m}}$

(c) $t_{\text{flight}} = \dfrac{2v_0\sin\theta}{g} = \dfrac{2\times 630\times\sin(0.5)}{9.8} = \underline{1.1\text{ s}}$

4-47. $x_{max} = v_0^2\sin 2\theta/g$. Let the target distance be **x**

$x - 180 = v_0^2\sin 14.667°/g$ (i)

$x + 120 = v_0^2\sin 15.167°/g$ (ii)

Dividing (i) by (ii) gives

$$\frac{x-180}{x+120} = \frac{\sin 14.667°}{\sin 15.167°} \Rightarrow (x-180)\sin 15.167° = (x+120)\sin 14.667°$$

$$x = \frac{77.48}{0.00844} = 9180\text{ m}$$

From (i) 9180 m = 180 m = v_0^2 sin 14.667°/g

$$v_0^2 = 3.483 \times 10^5\ \text{m}^2/\text{s}^2$$

$$\sin 2\theta = \frac{xg}{v_0^2} = \frac{9180 \times 9.8}{3.48 \times 10^5} = 0.23$$

$\theta = 1/2\ \sin^{-1} 0.23 = \underline{7.49° = 7°29'}$

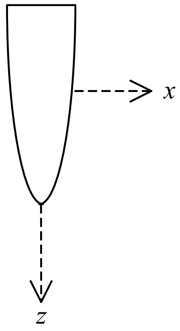
4-48. $x_{max} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g}$, $y_{max} = \dfrac{v_0^2 \sin^2\theta}{2g}$. Dividing the second equation by the first gives $\dfrac{y_{max}}{x_{max}} = \dfrac{\sin^2\theta}{4\sin\theta\cos\theta} = \dfrac{\tan\theta}{4}$, from which we get $\underline{\theta = \tan^{-1}\left(\dfrac{4y_{max}}{x_{max}}\right) = \tan^{-1}\left[\dfrac{4(2.5)}{5}\right] = 63.4°.}$

$$\underline{v_0 = \frac{\sqrt{2gy_{max}}}{\sin\theta} = \frac{\sqrt{2(9.81\ \text{m/s}^2)(2.5\ \text{m})}}{\sin 63.4°} = 7.8\ \text{m/s.}}$$

†4-49. Consider the motion of two of the projectiles. Call their positions (x_1, y_1) and (x_2, y_2) and assume that they were launched with elevation angles θ_1 and θ_2, respectively. The positions are given by $x_1 = v_0 \cos\theta_1 t,\ y_1 = v_0 \sin\theta_1 t - \dfrac{gt^2}{2}$ and $x_2 = v_0 \cos\theta_2 t,\ y_2 = v_0 \sin\theta_2 t - \dfrac{gt^2}{2}$. For the projectiles to collide, they must have the same positions at the same time: $x_1 = x_2$ and $y_1 = y_2$. This gives the following pair of equations: $v_0 \cos\theta_1 t = v_0 \cos\theta_2 t,\ v_0 \sin\theta_1 t - \dfrac{gt^2}{2} = v_0 \sin\theta_2 t - \dfrac{gt^2}{2}$. Canceling terms gives $\cos\theta_1 = \cos\theta_2,\ \sin\theta_1 = \sin\theta_2$. Dividing the second equation by the first gives $\tan\theta_1 = \tan\theta_2$ as the requirement for the projectiles to collide. Since we assumed that the projectiles were launched at different angles, we arrive at a contradiction, and we conclude that the individual projectiles will never collide during their flight.

4-50. Neglecting air resistance, the net force on the projectile is always $-mg\mathbf{j}$ (taking the $+y$ direction to point up). If the motion is instantaneously uniform circular motion at the top of the trajectory, then the magnitude of the centripetal force must be mg, and the centripetal acceleration must be g. Thus $\dfrac{v_{0x}^2}{r} = g$, which gives $\underline{r = \dfrac{v_{0x}^2}{g}}$ for the radius of the osculating circle.

†4-51. Take the x and z directions to point as shown. (The y direction is up, out of the page.) The projectile has two horizontal components of velocity: $\mathbf{v}_{horizontal} = v_0 \cos\theta\mathbf{i} + v_S\mathbf{k}$, where v_0 is the muzzle speed of the projectile, θ is the elevation angle of the projectile, and v_S is the speed of the ship. Neglecting air resistance, the flight time for the projectile is $t_{flight} = \dfrac{2v_0 \sin\theta}{g} = \dfrac{2(720\ \text{m/s})(\sin 30°)}{9.81\ \text{m/s}^2} = 73.5\ \text{s}$. The speed of the ship is 45 km/h = 12.5 m/s, so the net horizontal displacement of the projectile is $\mathbf{r} = \mathbf{v}_{horizontal} t_{flight}$



$= [(720\ \text{m/s})(\cos 30°)\mathbf{i} + (12.5\ \text{m/s})\mathbf{k}](73.5\ \text{s}) = (45.8 \times 10^3\ \text{m})\mathbf{i} + (918\ \text{m})\mathbf{k}$. The total horizontal range is $r = \sqrt{r_x^2 + r_z^2} = 45.8$ km. The change in the total range due to the ship's motion is negligible to three significant digits; however, failure to compensate for the ship's forward speed would cause the projectile to miss its target by nearly 1 km.

4-52. $x = v_0 t \cos\theta$ (i) $z = v_0 t \sin\theta - \frac{1}{2} g t^2$ (ii)

$t = \frac{x}{v_0 \cos\theta}$ [from (i)] (iii). Substituting (iii) into (ii) gives $z = x \tan\theta - \frac{1}{2}\frac{gx^2}{v_0^2 \cos^2\theta}$

$13 = 50 \tan\theta - \frac{1}{2}\frac{(9.8)(50)^2}{25^2}\sec^2\theta.$

$13 = 50 \tan\theta - 19.6(1 + \tan^2\theta)$

$19.6 \tan^2\theta - 50 \tan\theta + 32.6 = 0$

This equation has imaginary roots, so stone cannot hit the window.

$$z = 50 \tan\theta - 19.6(1 + \tan^2\theta)$$

Let $y = \tan\theta$, $z = -19.6y^2 + 50y = 19.6$

$\frac{dz}{d\theta} = \left(\frac{dz}{dy}\right)\left(\frac{dy}{d\theta}\right)$ and $\frac{dz}{dy} = -39.2y + 50$

At maximum, $dz/d\theta = 0$, so that $dz/dy = 0 = -39.2y + 50$

$y = 50/39.2 = 1.28 = \tan\theta$

$z = 50(1.28) - 19.6(1 + 1.28^2) = \underline{12.3\text{ m}}$

†4-53. From the diagram

$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$ (i)

In the xy-frame

$x = v_{0x} t$ (ii)

$y = v_{0y} t - \frac{1}{2} gt^2$ (iii)

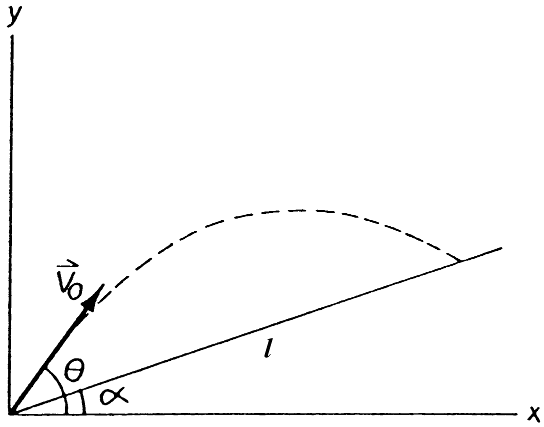
Also

$y = l \sin\alpha$

$x = l \cos\alpha$ (iv)

When the projectile hits the incline

$y = l \sin\alpha = v_{0y}t - \frac{1}{2} gt^2$ (v)



from (ii) $t = x/v_{0x} = l \cos\alpha / v_{0x}$ (vi)

Substitute t from (vi) in equation (v) to get the range as measured along the incline.

$$l = \frac{1}{\sin\alpha}\left\{v_0 \sin\theta \bullet \frac{l \cos\alpha}{v_0 \cos\theta} - \frac{1}{2} g \frac{l^2 \cos^2\alpha}{v_0^2 \cos^2\theta}\right\}$$

Solving for l, gives

$$l = \frac{2v_0^2}{g \cos\alpha}\left\{\cos\theta \sin\theta - \tan\alpha \cos^2\theta\right\}$$

$$\boxed{l = \frac{2v_0^2 \cos^2\theta}{g \cos\alpha}\left(\tan\theta - \tan\alpha\right)}$$

l will be maximum when $\frac{dl}{d\alpha} = 0$

$$\frac{dl}{d\theta} = \frac{2v_0^2}{g \cos\alpha}\left(\cos 2\theta + (\tan\alpha)\, 2 \cos\theta \sin\theta\right) = 0$$

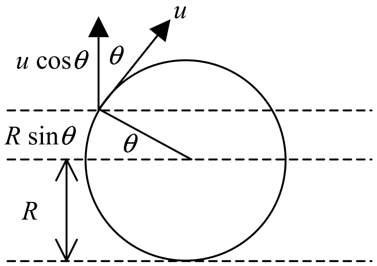
or $\cos 2\theta = -\tan\alpha \sin 2\theta$, $\cot 2\theta = -\tan\alpha = \cot(\alpha \pm \pi/2)$

Thus $\boxed{\theta = \frac{1}{2}(\alpha \pm \pi/2)}$

4-54. This problem is almost identical to 4-12. Follow the procedure given there. Using the slightly different numerical data for this problem leads to these equations for the position of the receiver and the position of the ball relative to the quarterback after the ball is thrown: $\mathbf{r}_R = (16 + 8t)\mathbf{i} + 15\,\mathbf{j}$, $\mathbf{r}_B = (20\sin\theta)t\mathbf{i} + (20\cos\theta)t\mathbf{j}$. The condition $\mathbf{r}_B = \mathbf{r}_R$ gives this pair of equations: $(20\sin\theta)t = 16 + 8t$; $(20\cos\theta)t = 15$. Use the second equation to get $t = \dfrac{15}{20\cos\theta} = \dfrac{3}{4\cos\theta}$. Substitute this into the first equation to get $20\sin\theta\left(\dfrac{4}{3\cos\theta}\right) = 16 + 8\left(\dfrac{4}{3\cos\theta}\right)$. Simplifying gives $\underline{15\sin\theta - 16\cos\theta = 6}$. Solving the transcendental equation gives 62.7°, or $\theta = \underline{63°}$ to two digits. The flight time for the ball is $t = \dfrac{3}{4\cos 62.7°} = 1.64$ s.

Call the elevation angle α. To find this angle, use the fact that we know both the range and the flight time. In terms of the initial speed of the ball and the angle α, we have $r = \dfrac{2v_0^2 \sin\alpha\cos\alpha}{g}$; $t = \dfrac{2v_0\sin\alpha}{g}$. Solve the second equation for v_0 and substitute that expression into the first equation: $r = 2\left(\dfrac{gt}{2\sin\alpha}\right)^2\left(\dfrac{\sin\alpha\cos\alpha}{g}\right) = \dfrac{gt^2\cos\alpha}{2\sin\alpha} = \dfrac{gt^2}{2\tan\alpha}$. The range can also be written in terms of v_B and t: $r = v_B t$, so we have $v_B t = \dfrac{gt^2}{2\tan\alpha}$, from which we get $\alpha = \tan^{-1}\left(\dfrac{gt}{2v_B}\right) = \tan^{-1}\left[\dfrac{(9.81\text{ m/s}^2)(1.64\text{ s})}{2(20\text{ m/s})}\right] = 21.9°$, or $\alpha = \underline{22°}$.

†4-55. The total instantaneous velocity of a point on the rim of the tire is the vector sum of the forward velocity of the tractor and the tangential velocity of the point, $\mathbf{v}_{tot} = \mathbf{v}_{tan} + \mathbf{u}$. This total instantaneous velocity must be zero when the point is on the road ($\theta = 270°$), or the tire will be slipping. Thus the tangential speed of a point on the rim must be equal to the forward speed u of the tractor.



In the reference frame of the tractor, the horizontal component of velocity of a glob is always zero and the vertical component of the velocity is $v_y = u\cos\theta$, which becomes the y component of the initial velocity v_{0y} of a launched glob. The maximum height above the launch point is $y_{max} = \dfrac{v_{0y}^2}{2g} = \dfrac{u^2\cos^2\theta}{2g}$. To find the maximum height above the ground, we must add the height of the launch point above the ground, which is $R + R\sin\theta$ as shown in the diagram. So the height above the ground is $h = \underline{R(1 + \sin\theta) + \dfrac{u^2\cos^2\theta}{2g}}$.

The maximum height depends on θ. To find the maximum possible height reached by a glob, differentiate h with respect to θ and set the derivative equal to zero:

$\dfrac{dh}{d\theta} = R\cos\theta - \dfrac{u^2\cos\theta\sin\theta}{g} = 0$, from which we get $\sin\theta_{max} = \dfrac{gR}{u^2}$. For R = 0.80 m and u = 30 km/h = 8.33 m/s, this gives θ_{max} = 6.49°, from which we get

$$h = (0.80\text{ m})(1+\sin 6.49°) + \frac{(8.33\text{ m/s})^2(\cos 6.49°)^2}{2(9.81\text{ m/s}^2)} = \underline{4.4\text{ m}}.$$

4-56. 40 km/h = 11.11 m/s

(a) The equation for the projectile is

$$(700\cos\theta)t\,\mathbf{i} + \left[(700\sin\theta)\,t - \frac{1}{2}gt^2\right]\mathbf{j} = \mathbf{r}_{\text{proj}}$$

The equation for the ship is $(1500 - 11.11t)\,\mathbf{i} = \mathbf{r}_{\text{ship}}$

When the projectile hits the ship, the coefficient of $\mathbf{j} = 0$, so

$$(700\sin\theta)\,t - \frac{1}{2}gt^2 = 0 \qquad \text{(i)}$$

Also, the coefficients of $\mathbf{i}$ must be equal, so

$(700\cos\theta)\,t = 15{,}000 - 11.11t$

$$t = \frac{15{,}000}{700\cos\theta + 11.11}$$

Substituting this into (i) gives

$$700\sin\theta = \frac{1}{2}gt = \frac{1}{2}g\,\frac{15{,}000}{(700\cos\theta + 11.11)}$$

$\sin\theta\,(700\cos\theta + 11.11) = 105$

An *iterated* solution gives $\underline{\theta \approx 8.6°}$

(b) The time interval is $t = \dfrac{15{,}000}{(700\cos 8.6° + 11.11)} = \underline{21.3\,\text{s}}$

4-57. If the ship does not move, the time of flight is given by $t_{\text{flight}} = 2v_0\sin\theta/g$

θ is given by $x_{\text{max}} = 17{,}000\text{ m} = v_0^2\sin 2\theta/g = 700^2\sin 2\theta/9.81$

$$\sin 2\theta = \frac{9.81\times 17{,}000}{700^2} = 0.34$$

$$\theta = \frac{1}{2}\sin^{-1}(0.34) = 9.94°$$

$$t_{\text{flight}} = \left(\frac{2\times 700\text{ m}\times\sin 9.94°}{9.81}\right)\text{s} = 24.67\text{ s}$$

In this time the ship would have moved 8.33 m/s × 24.67 s = 205.5 m.

Thus the new x_{max} (by the Pythagorean theorem) is

$$\sqrt{17{,}000^2 + 205.4^2} = 17{,}001\text{ m}$$

$$x_{\text{max}} = 17001\text{ m} = v_0^2\sin 2\theta/g \geq \theta = \frac{1}{2}\sin^{-1}\left(\frac{17001\times 9.8}{700^2}\right) = 9.94°$$

so the time of the flight is still the same, so the projectile will arrive there when the ship does.

$\underline{\theta = 9.94° = 9°56'}$

4-58. Let ω stand for the angular speed of the disk. Then $v = \underline{\omega R = (32.5 \text{ rad/s})(4.0 \text{ cm})} = \underline{130 \text{ cm/s}}$.

The rotation rate in rev/min is $\omega = (32.5 \text{ rad/s})(1 \text{ rev}/2\pi \text{ radians})(60 \text{ s/min}) = \underline{310 \text{ rev/min}}$.

†4-59. $a = g = \dfrac{v^2}{R} \Rightarrow v = \sqrt{gR} = \sqrt{(9.81 \text{ m/s}^2)(200 \text{ m})} = 44.3 \text{ m/s}$. Use ω as defined in the solution to 4-58 to find the number of revolutions per minute required to give this acceleration: $\omega = \underline{\dfrac{v}{R} = \dfrac{44.3 \text{ m/s}}{200 \text{ m}} \times \dfrac{60 \text{ s}}{\text{min}} \times \dfrac{1 \text{ rev}}{2\pi \text{ radians}} = 2.1 \text{ rev/min}}$.

4-60. $v = 3.0 \text{ m/s} \Rightarrow \omega = \dfrac{v}{R} = \dfrac{2v}{D}$, where D = diameter of hole. For $D = 3$ mm,

$$\omega = \underline{\frac{2(3.0 \text{ m/s})}{3 \times 10^{-3} \text{ m}} \times \frac{60 \text{ s}}{\text{min}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 1.9 \times 10^4 \text{ rev/min. For } D = 25 \text{ mm,}}$$

$$\omega = \underline{\frac{2(3.0 \text{ m/s})}{25 \times 10^{-3} \text{ m}} \times \frac{60 \text{ s}}{\text{min}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 2.3 \times 10^3 \text{ rev/min.}}$$

†4-61. Let T stand for the time for one complete orbit. $v = \dfrac{2\pi R}{T} \Rightarrow a = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$. For an orbit with a radius of 6500 km and $T = 87$ min, this gives $a = \underline{\dfrac{4\pi^2 (6500 \times 10^3 \text{ m})}{(87 \text{ min})^2 (60 \text{ s/min})^2} = 9.4 \text{ m/s}^2}$.

4-62. $v = \underline{\omega R = (45 \text{ rad/s})(0.80 \text{ m}) = 36 \text{ m/s}}$.

†4-63. $a = \dfrac{v^2}{r}$. Let f stand for the number of rotations per second. Then $v = 2\pi r f = 2\pi \times 0.1 \text{ m} \times 1000/\text{s}$ $= 200\pi$ m/s. $a = \dfrac{v^2}{r} = \dfrac{(200\pi \text{ m/s})^2}{(0.10 \text{ m})} = \underline{3.95 \times 10^6 \text{ m/s}^2}$. This is $= (3.95 \times 10^6 \text{ m/s}^2)/(9.81 \text{ m/s}^2)\, g$ $= \underline{4.0 \times 10^5\, g}$.

4-64. 1 rev. of blade = $\pi \times 0.2 = 0.63$ m.

Turning (7000/60) rev/sec, the speed is $v_{\text{tip}} = 0.63 \times \dfrac{7000}{60} = \underline{73.3 \text{ m/s}}$

The centripetal acceleration is $a_c = v_{tip}^2/r = (73.3)^2/0.1 = \underline{5.4 \times 10^4 \text{ m/s}^2}$

4-65. $v = 0.9999995\, c = 0.9999995 \times 2.998 \times 10^8 \text{ m/s} = 2.998 \times 10^8 \text{ m/s}$

$r = 1.0 \text{ km} = 1000 \text{ m}$

$a = v^2/r = (2.998 \times 10^8 \text{ m/s})^2/1000 \text{ m} = \underline{8.99 \times 10^{13} \text{ m/s}^2 = 9.16 \times 10^{12} \text{ gee}}$

4-66. $v = \omega R = (33.33 \text{ rev/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/rev})(15 \text{ cm}) = \underline{52 \text{ cm/s}}$.

4-67. $r = 1.50 \times 10^{11}$ m

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 1.50 \times 10^{11} \text{ m}}{365.25 \times 24 \times 3600} = 2.99 \times 10^4 \text{ m/s}$$

$$a_c = \frac{(2.99 \times 10^4)^2}{1.50 \times 10^{11}} \text{ m/s}^2 = \underline{5.9 \times 10^{-3} \text{ m/s}^2}$$

4-68. $v = 95$ km/h = 26.4 m/s. This is also the tangential speed of a point on the rim of the wheel, so

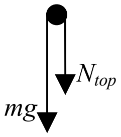
$$a = \frac{v^2}{r} = \frac{(26.4 \text{ m/s})^2}{(0.32 \text{ m})} = \underline{2.2 \times 10^3 \text{ m/s}^2}.$$

4-69. $v = \dfrac{2\pi R}{T} \Rightarrow a = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$. T = 23h 56 min = 86160 s. At the equator,

$$a_{eq} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(6.37 \times 10^6\,\text{m})}{(86160\ \text{s})^2} = 3.39 \times 10^{-2}\ \text{m/s}^2.$$ At a latitude of 45°, the radius is the equatorial radius multiplied by cos 45°, so $a_{45} = a_{eq}\cos 45° = 2.40 \times 10^{-2}\ \text{m/s}^2$.

4-70. At the top, $a_{top} = \dfrac{v_{top}^2}{r}$. 350 km/h = 97.2 m/s. $a_{top} = \dfrac{(97.2\ \text{m/s})^2}{500\ \text{m}} = 18.9\ \text{m/s}^2$. At the bottom,

$a_{bottom} = \dfrac{v_{bottom}^2}{r}$. 620 km/h = 172 m/s. $a_{bottom} = \dfrac{(172\ \text{m/s})^2}{500\ \text{m}} = 59.3\ \text{m/s}^2$.

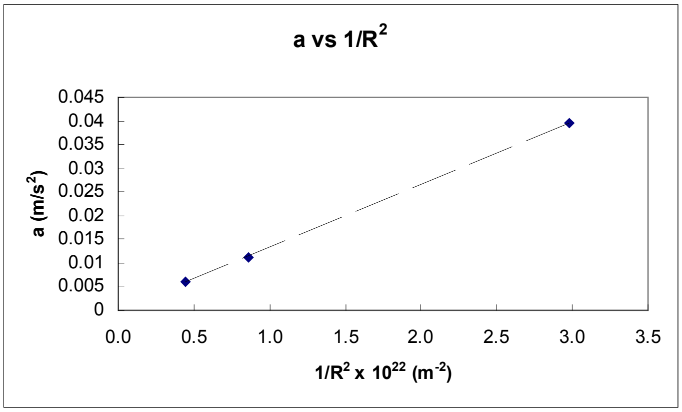


At the top, consider the free body diagram for the pilot, who is upside down so the normal force exerted by the seat points down. Taking the + direction to point down, Newton's Second Law says $N_{top} + mg = ma_{top}$. $\dfrac{N_{top}}{m}$ = acceleration the pilot feels = $a_{top} - g$ = (18.9 – 9.81) m/s^2 = 9.09 m/s^2. At the bottom, the pilot is upright so the normal force points up. Taking the + direction to point up, Newton's Second Law gives $N_{bottom} - mg = ma_{top}$. Now $\dfrac{N_{bottom}}{m} = g + a_{bottom}$ = (9.81 + 59.3) m/s^2 = 69.1 m/s^2.

†4-71. In the table shown, the acceleration was calculated using $a = \dfrac{4\pi^2 R}{T^2}$.

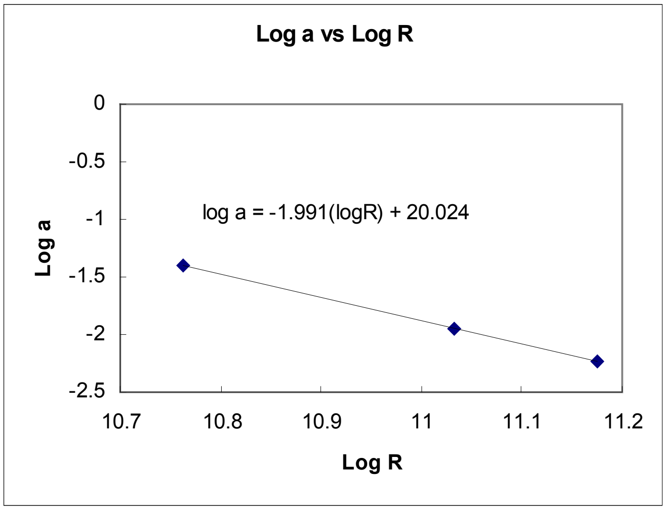
Planet	R (m)	T (yr)	a (m/s^2)	1/R^2 (m^{-2})
Mercury	5.79E+10	0.241	0.0395	2.98E-22
Venus	1.08E+11	0.615	0.0113	8.57E-23
Earth	1.50E+11	1.000	0.00595	4.44E-23

One way to see if the centripetal acceleration is inversely proportional to R^2 is to plot a graph of a *vs* $1/R^2$. The resulting plot is a straight line, proving the proportionality. (In the graph, the values of $1/R^2$ have been multiplied by 10^{22} to eliminate large negative exponents for the x axis values.)



Another approach to the analysis is to calculate the log of a and R and plot a graph of log a vs log R. If a is inversely proportional to $1/R^2$, then this graph should be a straight line with a slope of –2.

log a	log R
−1.4034	10.76268
−1.94692	11.03342
−2.22548	11.17609



The graph was plotted in a spreadsheet, which was used to find the equation for the resulting line. The slope turns out to be almost exactly −2, again proving the desired proportionality.

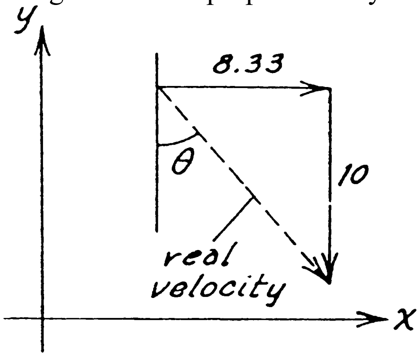
4-72. 30 km/h = 8.33 m/s

$\mathbf{v} = \mathbf{v}' + \mathbf{V}$

$= -10\,\mathbf{j} + 8.33\,\mathbf{i}$

$v = \sqrt{10^2 + 8.33^2} = \underline{13.0 \text{ m/s}}$

The angle θ is $\theta = \tan^{-1}\left(\dfrac{8.33}{10}\right) = \underline{40°}$



4-73. $\mathbf{v}_{ground} = \mathbf{v}_{walk} + \mathbf{v}_{passenger}$. If the velocity of the walkway and the velocity of the passenger are in the same direction, then $\underline{v_{ground} = v_{walk} + v_{passenger} = 1.5 \text{ m/s} + 4.0 \text{ m/s} = 5.5 \text{ m/s}}$. If the passenger is walking in the opposite direction from the walkway's velocity, the speed of the passenger is $\underline{v_{ground} = |v_{walk} - v_{passenger}| = |1.5 \text{ m/s} - 4.0 \text{ m/s}| = 2.5 \text{ m/s}}$.

4-74. $\mathbf{v}_{\text{rain}} = -10\,\mathbf{j}$ m/s; $\mathbf{v}_{\text{car}} = 25\,\mathbf{i}$ m/s

$\mathbf{v}' = \mathbf{v}_{\text{rain}} - \mathbf{v}_{\text{car}} = -25\,\mathbf{i} - 10\,\mathbf{j}$ m/s

$v' = \sqrt{25^2 + 10^2} = \underline{26.9 \text{ m/s}}$

$\theta = \tan^{-1} 25/10 = \underline{68°}$

4-75. $\mathbf{v}_{\text{ship}} = 13\,\mathbf{i}$ m/s

$\mathbf{v}_{\text{muzzle}} = v\cos\theta\,\mathbf{i} + v\sin\theta\,\mathbf{j} = 660\cos 20°\,\mathbf{i} + 660\sin 20°\,\mathbf{j}$

$= 620\,\mathbf{i} + 226\,\mathbf{j}$ m/s

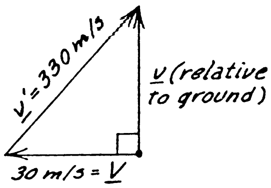
$\mathbf{v}_{\text{shot}} = \mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{muzzle}} = (620 + 13)\,\mathbf{i} + 226\,\mathbf{j}$ m/s $= \underline{633\,\mathbf{i} + 226\,\mathbf{j} \text{ m/s}}$

(magnitude: $v_{\text{shot}} = 672$ m/s, $\theta = \tan^{-1} 226/633 = 19.6°$)

4-76. $\mathbf{v} = \mathbf{v}' + \mathbf{V}$

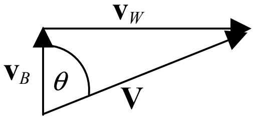
By the Pythagorean theorem:

$v = \sqrt{330^2 - 30^2} = \underline{329 \text{ m/s}}$



4-77. $\mathbf{V} = \mathbf{v}_B + \mathbf{v}_W \Rightarrow V = \sqrt{v_B^2 + v_W^2} = \sqrt{(1.5\text{ m/s})^2 + (12\text{ m/s})^2}$

$V = \underline{12\text{ m/s}}$. $\theta = \tan^{-1}\dfrac{v_W}{v_B} = \tan^{-1}\dfrac{12}{1.5} = \underline{83^\circ}$.



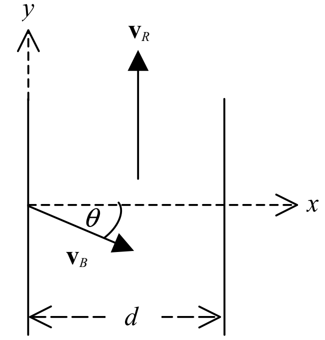
4-78. The speeds add going downstream, so $v_{down} = 6.0$ km/h. In 40 min, you can go $d = 6.0\text{ km/h} \times 40\text{ min} \times 1\text{ h} / 60\text{ min} = \underline{4.0\text{ km}}$. Upstream, the speed of the current must be subtracted from the speed of the boat, so $v_{up} = 1.0$ km/h. The time to make the return trip of 4.0 km is $t = \dfrac{d}{v_{up}} = \dfrac{4\text{ km}}{1\text{ km/h}} = \underline{4.0\text{ h}}$.

†4-79. $V = \left|\mathbf{v}_{train} + \mathbf{v}_{flea}\right|$, where $\mathbf{v}_{flea}$ is the velocity of the flea relative to the train. The cat is moving backwards relative to the train and the flea is moving backwards relative to the cat. The velocity of the flea relative to the train is 0.50 m/s – 0.10 m/s = 0.40 m/s backwards relative to the train. So the flea's speed relative to the ground is $V = \left|5.00\text{ m/s} - 0.40\text{ m/s}\right| = \underline{4.60\text{ m/s}}$.

4-80. The position of any point on the shore is given by $\mathbf{r} = d\mathbf{i} + y\mathbf{j}$, where y is the displacement (positive or negative) along the y axis of the landing point on the opposite shore. The velocity of the boat relative to the shore is $\mathbf{V} = v_B\cos\theta\mathbf{i} + (v_R - v_B\sin\theta)\mathbf{j}$. The time to reach any point on the opposite shore is given by $\mathbf{r} = \mathbf{V}t$, or $d\mathbf{i} + y\mathbf{j} = (v_B\cos\theta)t\mathbf{i} + (v_R - v_B\sin\theta)t\mathbf{j}$. This is equivalent to two equations:

$$d = (v_B\cos\theta)t$$

$$y = (v_R - v_B\sin\theta)t$$



To reach a point directly opposite the launch point, $y = 0$, which means $\sin\theta = \dfrac{v_R}{v_B}$. This describes a right triangle with a hypotenuse v_B and an opposite side v_R, so we know the adjacent side is $\sqrt{v_B^2 - v_R^2}$. That means $\cos\theta = \dfrac{\sqrt{v_B^2 - v_R^2}}{v_B}$. Use this in the first equation to get t:

$$\underline{t = \frac{d}{v_B\cos\theta} = \frac{d}{\sqrt{v_B^2 - v_R^2}}}.$$

To find the time to reach any point on the opposite shore, begin with $\cos\theta = \dfrac{d}{v_B t}$, from which we deduce $\sin\theta = \dfrac{\sqrt{(v_B t)^2 - d^2}}{v_B t}$. Substitute this into the y equation to get

$$y = \left(v_R - v_B\frac{\sqrt{(v_B t)^2 - d^2}}{v_B t}\right)t = v_R t - \sqrt{(v_B t)^2 - d^2}.$$ Rearrange to get $\sqrt{(v_B t)^2 - d^2} = v_R t - y$.

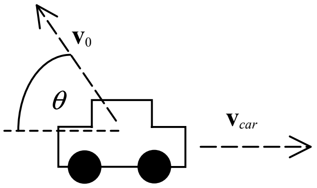
Square both sides and rearrange: $(v_B^2 - v_R^2)t^2 + 2v_R y t - (d^2 + y^2) = 0$. The roots of this equation are $t = \dfrac{-2v_R y \pm \sqrt{(2v_R y)^2 + 4(v_B^2 - v_R^2)(d^2 + y^2)}}{2(v_B^2 - v_R^2)}$. The time must be positive, so choose the +

sign in front of the square root. Rearrange and simplify to get $t = \dfrac{\sqrt{(v_B^2 - v_R^2)d^2 + v_B^2 y^2} - v_R y}{(v_B^2 - v_R^2)}$. As a quick check, set $y = 0$ and we get the same result we had in the first part of the problem.

4-81. $\mathbf{v}_{esc} = \dfrac{30\text{ cm}}{1.5\text{ s}}\mathbf{i} + \dfrac{20\text{ cm}}{1.5\text{ s}}\mathbf{j} = (20\text{ cm/s})\mathbf{i} + (13.3\text{ cm/s})\mathbf{j}$. The rider is walking up with a velocity $\mathbf{v}_{rider} = \dfrac{30\text{ cm}}{1.0\text{ s}}\mathbf{i} + \dfrac{20\text{ cm}}{1.0\text{ s}}\mathbf{j} = (30\text{ cm/s})\mathbf{i} + (20\text{ cm/s})\mathbf{j}$. The rider's total velocity relative to the floor is

$\mathbf{V} = \mathbf{v}_{esc} + \mathbf{v}_{rider} = \dfrac{30\text{ cm}}{1.0\text{ s}}\mathbf{i} + \dfrac{20\text{ cm}}{1.0\text{ s}}\mathbf{j} = (20\text{ cm/s} + 30\text{ cm/s})\mathbf{i} + (13.3\text{ cm/s} + 20\text{ cm/s})\mathbf{j} =$ $(50\text{ cm/s})\mathbf{i} + (33.3\text{ cm/s})\mathbf{j}$. The speed is $v_{floor} = \sqrt{(50\text{ cm/s})^2 + (33.3\text{ cm/s})^2} = 60\text{ cm/s}$. The direction of the velocity relative to the floor is $\theta = \tan^{-1}\dfrac{33.3}{50} = 34°$.

4-82. $\mathbf{v}_0$ = initial velocity relative to the car. If the projectile is observed to move straight up and down relative to an observer on the ground, then the horizontal component of the projectile's initial velocity must have the same magnitude as the velocity of the car and point in the opposite direction. Thus



$v_0 \cos\theta = v_{car} \Rightarrow \theta = \cos^{-1}\dfrac{v_{car}}{v_0} = \cos^{-1}\dfrac{30}{50} = 53.1°$. The maximum height reached is

$$y_{max} = \frac{v_{0y}^2}{2g} = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(50\text{ m/s})\sin 53.1°]^2}{2(9.81\text{ m/s}^2)} = 81\text{ m}.$$

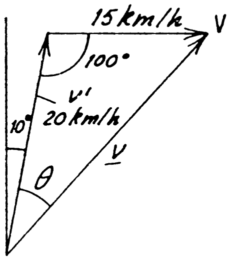
4-83. $\mathbf{v} = \mathbf{v}' + \mathbf{V}$

$v = \sqrt{20^2 + 15^2 - 2(20)(15)\cos 100°} = 27\text{ km/h}$

$15/\sin\theta = 27/\sin 100°$

$\sin\theta = \dfrac{15}{27}\sin 100° = 0.547$

$\theta = 33°$



4-84. $\mathbf{v} = \mathbf{v}' + \mathbf{V}$,

where $\mathbf{v}'$ = velocity of wind relative to boat

$\mathbf{V}$ = velocity of boat

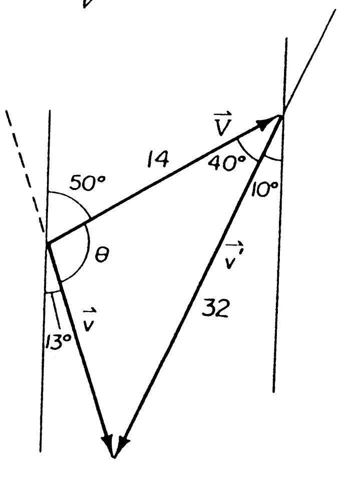
$\mathbf{v}$ = velocity of wind relative to ground

$v = \sqrt{(14)^2 + (32)^2 - 2(14)(32)\cos 40°} = 23.1\text{ km/h}$

$\dfrac{\sin\theta}{32} = \dfrac{\sin 40°}{23.1}$

$\theta = \sin^{-1}\left(\dfrac{32}{23.1}\sin 40°\right) = 117°$

Wind is coming from 13° W of N



4-85. Time of descent $t = \dfrac{1500}{0.5} = 3000$ s

(a) Horizontal distance travelled before reaching the ground,

$$x = \left(\frac{60 \times 10^3}{60 \times 60}\right)(3000) = \underline{50 \times 10^3 \text{ m}}$$

(b) Relative velocity with respect to still air 60 – 20 = 40 km/h

Therefore $x = \dfrac{40 \times 10^3}{60 \times 60} \times 3000 = \underline{33 \times 10^3 \text{ m}}$

In the upwind direction, the relative velocity is 60 + 20 = 80 km/h, and

$$x = \frac{80 \times 10^3}{60 \times 60} \times (3000) = \underline{67 \times 10^3 \text{ m}}$$

4-86. $\sin\theta/50 = \sin 135°/250$

$\theta = \sin^{-1}\left(\frac{1}{5}\sin 135°\right)$

$\theta = \sin^{-1}(0.1414) = 8°$

$\alpha = 180° - (135° + 8°) = 37°$

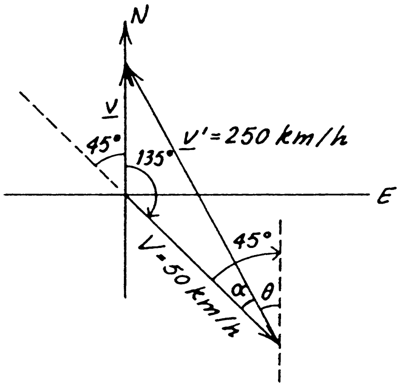
Pilot must point the plane

$(45° - 37°) = \underline{8° \text{ W of N}}$

$v/\sin\alpha = 250/\sin 135°$

$v = \sin 37° \times 250/\sin 135°$ km/h

$= \underline{213 \text{ km/h}}$



†4-87. $v = \sqrt{4.2^2 + 16^2 - 2(4.2)(16)\cos 70°}$

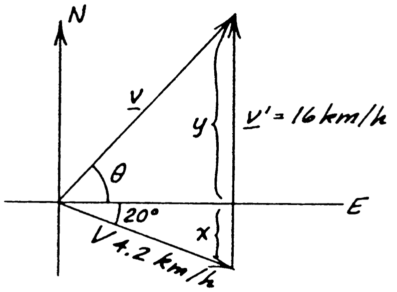
$= 15.1$ km/h

$x = 4.2 \sin 20° = 1.44$ km/h

$y = (16 - 1.44)$ km/h $= 14.56$ km/h

$\sin\theta = 14.56/v = 14.56/15.1$

$\theta = \sin^{-1}(0.96) = \underline{75°}$ (15° E of N)

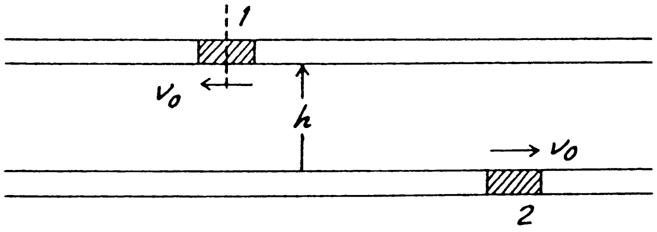


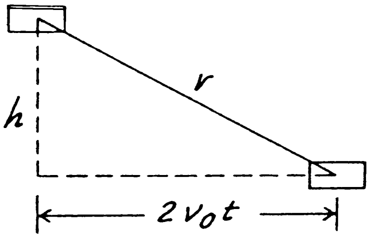
4-88. (a) average speed = $\Delta r_{\text{rel}}/\Delta t = 1$ km/2 min = 1 km/(1/30h) = $\underline{30 \text{ km/h}}$

(b) $\mathbf{v} = \mathbf{v}' + \mathbf{V}$ with $V = 90$ km/h; $v' = 30$ km/h

$v = (90 + 30)$ km/h $= \underline{120 \text{ km/h}}$

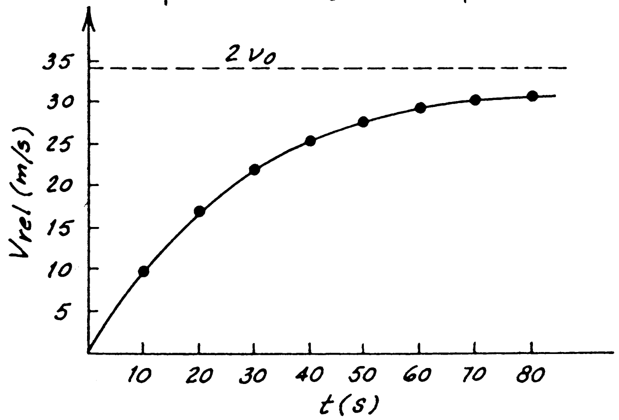
4-89. Let the origin be fixed on car 1. Then *in that frame*, car 2 moves at a speed of $2v_0$.





If they are "together" (at $t = 0$), then after time t, they are separated by

$$r = \sqrt{h^2 + (2v_0t)^2}$$



Then,

$$v_{\text{rel}} = \frac{dr}{dt} = \frac{d}{dt}\left(h^2 + 4v_0^2t^2\right)^{1/2}$$

$$= \frac{1}{2}\left(h^2 + 4v_0^2t^2\right)^{-1/2}\left(8v_0^2t\right)$$

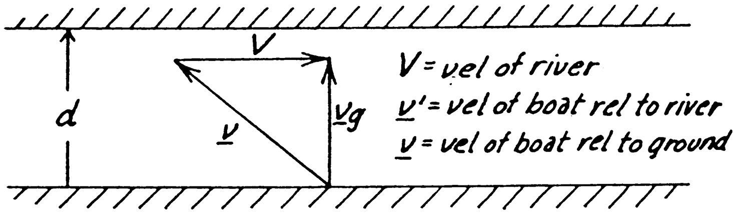
$$v_{\text{rel}} = \frac{4v_0^2t}{\left(h^2 + 4v_0^2t^2\right)^{1/2}}$$

$$= \frac{2v_0}{\left(\dfrac{h^2}{4v_0^2t^2} + 1\right)^{1/2}}$$

[Note: as $t \to \infty$, $v_{\text{rel}} \to 2v_0$]

time (s)	v_{rel} (m/s)
10	9.6
20	17.1
30	22.3
40	25.6
50	27.7
60	29.1
70	30.0
80	30.8

†4-90. $\mathbf{v}_g = \mathbf{v} + \mathbf{V}$



(a) $v_g^2 = \sqrt{v^2 - v^2}$

Total distance across and back is $2d$.

$t = \text{dist}/v_g = \underline{2d/\sqrt{v^2 - V^2}}$

(b) $v_{\text{up}} = v - V$

$v_{\text{down}} = v + V$

$t_{\text{up}} = d/(v - V)$; $t_{\text{down}} = d/(v + V)$

$t_{\text{tot}} = d/(v - V) + d/(v + V) \underline{= 2dv/(v^2 - V^2)}$

The trip up and back takes longer (the denominator is larger)

†4-91. The velocity of the AWACS relative to the ground is

$\mathbf{v}_{\text{AW}} = 150\ \mathbf{i} + 750\ \mathbf{j}$ km/h

Relative to this, the UFO has velocity

$\mathbf{v}_{\text{UFO}} = -950 \cos 45°\ \mathbf{i} = 950 \sin 45°\ \mathbf{j}$ km/h =

$672\ \mathbf{i} - 672\ \mathbf{j}$ km/h

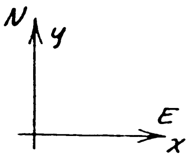
The velocity of the UFO relative to the ground is

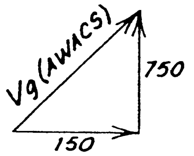
$$\mathbf{v}_{g(\text{UFO})} = (150\ \mathbf{i} + 750\ \mathbf{j}) + (-672\ \mathbf{i} - 672\ \mathbf{j}) \text{ km/h}$$

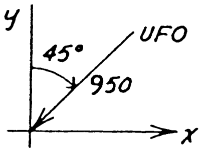
$$= 522\ \mathbf{i} + 78\ \mathbf{j} \text{ km/h}$$

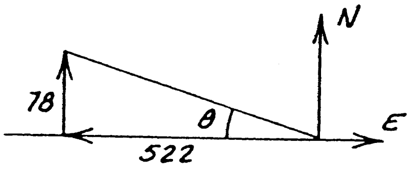
$$v_{g(\text{UFO})} = \sqrt{522^2 + 78^2} = \underline{528 \text{ km/h}}$$

The bearing is $\theta = \tan^{-1}\left(\frac{78}{522}\right) = \underline{8.5° \text{ N of W}}$



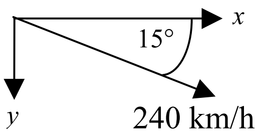






4-92. The average vertical velocity is $\mathbf{v}_y = -\frac{h}{t}\mathbf{j}$, where h is the vertical distance traveled in time t, and the positive direction is assumed to be up. $\underline{\mathbf{v}_y = -\frac{2340 \text{ m}}{3.5 \text{ h}}\mathbf{j} = (-669 \text{ m/h})\mathbf{j}}$. No information is given about the horizontal distance traveled, so there is no way to determine the average horizontal component of velocity. Since the speed is $\sqrt{v_x^2 + v_y^2}$, it's not possible to find the average speed from the information given.

4-93. Assume the airplane is gliding at a constant velocity, so its horizontal and vertical components of velocity are constant.

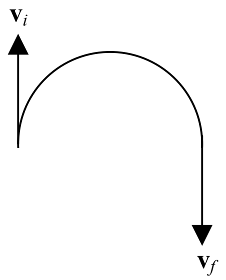


(a) Taking directions as shown in the diagram, $v_x = \underline{(240 \text{ km/h}) \cos 15° = 232 \text{ km/h}}$. The vertical component of velocity is $v_y \underline{= (240 \text{ km/h}) \sin 15° = 62.1 \text{ km/h}}$.

(b) The time to fall a distance $h = 2000$ m = 2 km is given by

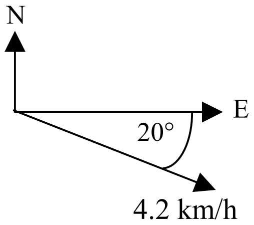
$$t = \frac{h}{v_y} = \frac{2 \text{ km}}{62.1 \text{ km/h}} = 0.0322 \text{ h} = \underline{1.9 \text{ min}}.$$

4-94. $$\underline{\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} = \frac{(-25 \text{ m/s})\mathbf{j} - (25 \text{ m/s})\mathbf{j}}{12 \text{ s}} = 0\mathbf{i} - (4.2 \text{ m/s}^2)\mathbf{j}}.$$



4-95. $v_E = \underline{(4.2\text{ km/h})\cos 20^\circ = 3.9\text{ km/h}}.$

$v_N = \underline{-(4.2\text{ km/h})\sin 20^\circ = -1.4\text{ km/h}}.$



4-96. $\mathbf{v} = (15\text{ km/h})\mathbf{i} + (15\text{ km/h})\mathbf{j} \Rightarrow v = \underline{\sqrt{(15\text{ km/h})^2 + (15\text{ km/h})^2} = (15\text{ km/h})\sqrt{2} = 21\text{ km/h}}.$ The direction is $\theta t = \tan^{-1}\dfrac{v_y}{v_x} = 45^\circ,$ or $\underline{45^\circ\text{ E of N}}$.

†4-97. (a) $\mathbf{r} = (6.0 + 2.0t^2)\mathbf{i} + (3.0 - 2.0t + 3.0t^2)\mathbf{j}.$ $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 4.0t\mathbf{i} + (-2.0 + 6.0t)\mathbf{j}.$ At $t = 2.0$ s, $\mathbf{v} = 8.0\mathbf{i} + 10\mathbf{j}$ m/s, so the speed is $v = \underline{\sqrt{v_x^2 + v_y^2} = \sqrt{(8.0\text{ m/s})^2 + (10\text{ m/s})^2} = 13\text{ m/s}}.$

(b) $\mathbf{a} = \dfrac{d\mathbf{v}}{dt} = 4.0\mathbf{i} + 6.0\mathbf{j}\text{ m/s}^2$ at all times. The magnitude of the acceleration is $\underline{a = \sqrt{a_x^2 + a_y^2} = \sqrt{(4.0\text{ m/s}^2)^2 + (6.0\text{ m/s}^2)^2} = 7.2\text{ m/s}^2}.$ The direction relative to the x axis is $\underline{\theta = \tan^{-1}\dfrac{v_y}{v_x} = \tan^{-1}\dfrac{6.0}{4.0} = 56.3^\circ}.$

4-98. (a) $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(1.5\text{ m})}{9.81\text{ m/s}^2}} = 0.553\text{ s}.$ Final result: $t = \underline{0.55\text{ s}}$.

(b) $x = \underline{v_0 t = (60\text{ m/s})(0.553\text{ s}) = 33\text{ m}}.$

4-99. (a) $y_{max} = \dfrac{v_0^2\sin^2\theta}{2g}.$ The maximum possible height would occur for $\theta = 90^\circ$, so

$y_{max} = \dfrac{(700\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = \underline{2.5\times 10^4\text{m, or 25 km}}.$

(b) $x_{max} = \dfrac{2v_0^2\sin\theta\cos\theta}{g}.$ The maximum horizontal range would occur for $\theta = 45^\circ$, so

$x_{max} = \dfrac{(700\text{ m/s})^2}{9.81\text{ m/s}^2} = \underline{5.0\times 10^4\text{ m, or 50 km}}.$

(c) It is not reasonable to ignore air resistance because at such speeds, air friction would be considerable. This is why the rocks do not rise to 25 km or cannot be thrown to distances approaching 50 km.

4-100. $x_{max} = \dfrac{2v_0^2\sin\theta\cos\theta}{g} \Rightarrow v_0 = \sqrt{\dfrac{gx_{max}}{2\sin\theta\cos\theta}}.$ For $\theta = 45^\circ$, $\sin\theta = \cos\theta = 1/\sqrt{2},$ so

$\underline{v_0 = \sqrt{(9.81\text{ m/s}^2)(180\text{ m})} = 42\text{ m/s}}.$

†4-101. $y = y_0 + v_{0y}t - \dfrac{gt^2}{2}$, $x = v_{0x}t$. When $y = 0$, $x = x_{max}$. For $\theta = 45°$, $v_{0x} = v_{0y} = v_0/\sqrt{2}$. Then the time to reach $y = 0$ is $t = \dfrac{\left(\dfrac{v_0}{\sqrt{2}}\right) + \sqrt{\dfrac{v_0^2}{2} + 2y_0 g}}{g} = \dfrac{v_0 + \sqrt{v_0^2 + 4y_0 g}}{g\sqrt{2}}$. (There is another value of t that solves the equation for $y = 0$, but it will be negative because the square root in the quadratic formula will be larger than v_0. Choose the positive value.) The horizontal range is $x_{max} = \dfrac{v_0}{\sqrt{2}}\left(\dfrac{v_0 + \sqrt{v_0^2 + 4y_0 g}}{g\sqrt{2}}\right) = \dfrac{v_0^2 + v_0\sqrt{v_0^2 + 4y_0 g}}{2g}$. Rearrange: $2gx_{max} - v_0^2 = v_0\sqrt{v_0^2 + 4y_0 g}$.

Square both sides: $4g^2x_{max}^2 - 4gx_{max}v_0^2 + v_0^4 = v_0^4 + 4gy_0v_0^2$. Cancel the fourth degree terms and the remaining common factor of 4 and solve for v_0:

$$v_0 = \sqrt{\frac{x_{max}^2 g}{x_{max} + y_0}} = \sqrt{\frac{(70.87\ \text{m})^2(9.81\ \text{m/s}^2)}{70.87\ \text{m} + 2.0\ \text{m}}} =$$

$\underline{26\ \text{m/s}}$.

4-102. For a horizontal initial velocity, the time for the water to fall a distance h is $t = \sqrt{\dfrac{2h}{g}}$ and the horizontal range is $x_{max} = v_0 t = v_0\sqrt{\dfrac{2h}{g}}$. This gives $v_0 = x_{max}\sqrt{\dfrac{g}{2h}} = (4.0\ \text{m})\sqrt{\dfrac{9.81\ \text{m/s}^2}{2(1.0\ \text{m})}} = 8.86$ m/s. Assuming that the speed of the water stays the same when the hose is pointed straight up gives $\underline{y_{max} = \dfrac{v_0^2}{2g} = \dfrac{(8.86\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 4.0\ \text{m}}$.

4-103. 90 km/h = 25 m/s. $\underline{a = \dfrac{v^2}{r} = \dfrac{(25\ \text{m/s})^2}{70\ \text{m}} = 8.9\ \text{m/s}^2}$.

4-104. Relative to the ground, the concrete has velocity

$v = \sqrt{2ax} = \sqrt{2 \times 9.8 \times 5}\ \text{m/s} = 9.90\ \text{m/s}$

The velocity of the car is 90 km/h = 25 m/s. If v' is the velocity of the concrete in the frame of the car, then

$\mathbf{v}' = \mathbf{v} - \mathbf{V} = -9.9\,\mathbf{j} - 25\,\mathbf{i}$ m/s

$\underline{v' = \sqrt{9.9^2 + 25^2} = 27\text{m/s}}$

Angle of impact = $\theta = \tan^{-1}\left(\dfrac{25}{9.9}\right) = \underline{68°}$

†4-105. (a) $V = \left|v_{car} - v_{truck}\right| = |90\ \text{km/h} - 60\ \text{km/h}| = 30\ \text{km/h} = 8.33\ \text{m/s}$.

(b) In the reference frame of the truck, the car has to travel a total distance of 90 m: It must travel 40 m to catch up to the truck, 10 m to pass the truck, and then travel another 40 m to get ahead of the truck. The time for the car to travel 90 m at 8.33 m/s is $t = \dfrac{90\ \text{m}}{8.33\ \text{m/s}} = \underline{10.8\ \text{s}}$.